CHAPTER – FACTORS & MULTIPLES

FACTORS & MULTIPLES

If a number a divides another number b exactly, we say that a is a **factor** of b. In this case, b is called a **multiple** of a.

Suppose, a number X is exactly divisible by Y,

i.e., $\frac{Y)\ X\ (q}{\text{Remainder} = 0}$

Then X = Y. q where Y (**divisor**) is a factor of X and X (**dividend**) is a multiple of Y.

$$\frac{X}{Y} = \frac{\text{Multiple}}{\text{Factor}} =$$

$$\text{quotient} = q = \frac{\text{Dividend}}{\text{Divisor}}$$

e.g., 6 is a factor of 18 and 18 is a multiples of 6.

$18 = 1 \times 3 \times 3 \times 2.$

So, 18 has the factors 1, 2, 3, 6, 9 and 18.

Similarly, 18 = 3 × 6 i.e., 3 multiples of 6.

So, divisor is the factor and dividend is the multiple.

HCF (Highest Common Factor) or G.C.D. (Greatest Common Divisor)

The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly and that is always smaller or equal to those numbers. So, H.C.F (Highest Common Factor). is the divisor. (or factor)

HCF or GCD of two or more numbers is the greatest number that divides each one of them exactly.

Methods of finding the H.C.F. of a given set of numbers

PRIME FACTORS

A factor of a given number is called a prime factor if this factor is a prime number.

EXAMPLE: The factors of 42 are 1, 2, 3, 6, 7, 14, 21 and 42. Out of these 2, 3 and 7 are prime numbers. Therefore, 2, 3 and 7 are the prime factors of 42.

1. PRIME FACTORIZATION METHOD:

Express each one given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.

PRIME FACTORIZATION METHOD : Suppose we have to find the H.C.F. of two or more numbers.

Step 1. Express each one of the given numbers as the product of prime factors.

Step 2. The product of terms containing least powers of common prime factors gives the H.C.F. of the given numbers.

Illustrations –1:

Find the H.C.F. of 540 and 1008

Solution`

Resolving each of the given numbers into prime factors, we get :

$540 = 2^2 \times 3^3 \times 5$

$1008 = 2^4 \times 3^2 \times 7$

$\therefore$ H.C.F. = Product of terms containing least powers of common prime factors

$= 2^2 \times 3^2 = (4 \times 9) = 36.$

2. LONG DIVISION METHOD : Suppose we to find the H.C.F. of two given numbers.

Step 1. Divide the larger number by smaller one.

Step 2. Divide the divisor by the remainder.

Step 3. Repeat the process of dividing the preceding divisor by the remainder last obtained, till remainder 0 is obtained.

Then, the last divisor is the required H.C.F.

Now, H.C.F. of three numbers = H.C.F. of [(H.C.F. of any two) and the third]

H.C.F. OF THREE GIVEN NUMBERS:

H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

Illustrations –2:

Find out the H.C.F of 3^5, 3^9 and 3^{14}.

Solution:

Here the based of each number is same (=3) but indices are different.

So, the required H.C.F. = number with the minimum index, ie., 3^5.

LEAST COMMON MULTIPLE (L.C.M.)

The least number which is exactly divisible by one of the given numbers is called their L.C.M.

Methods of finding the L.C.M of a given set of numbers.

1. FACTORIZATION METHOD

Each one of the given numbers is first resolved into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.

2. COMMON DIVISION METHOD (SHORT-CUT METHOD) OF FINDING L.C.M.

Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.

Illustrations –3:

Find out the L.C.M. of 4^5, 4^8, 4^{12} and 4^7.

Solution:

Here the base of each number is the same (= 4) but indices (or powers) are different.

So, the required L.C.M. = number with the maximum index, i.e., 4^{12}.

Illustrations –4:

Find the least number exactly divisible by 12, 15, 20 and 27.

Solution:

Required number = L.C.M. of 12, 15, 20, 27.

3	12 – 15 – 20 - 27
4	4 – 5 – 20 - 9
5	1 – 5 – 5 - 9
	1 – 1 – 1 - 9

$\therefore$ $L.C.M. = 3 \times 4 \times 5 \times 9 = 540.$

Hence, required number = 540.

PRODUCT OF TWO NUMBERS

H.C.F. of numbers × L.C.M. of numbers = Product of numbers

i.e., if the numbers are A and B then

(H.C.F. of A and B) × (L.C.M. of

A and B) = A × B

Illustrations –5:

H.C.F. and L.C.M. of two numbers are 16 and 240 respectively. If one of the numbers is 48, find the number.

Solution:

We know that, H.C.F × L.C.M. = Product of two numbers

$\therefore$ Second number = $\frac{16 \times 240}{480}$ i.e., 80.

DIFFERENCE BETWEEN H.C.F. AND L.C.M.

H.C.F. of x, y and z is the Highest Divisor which can exactly divide all of them

Co-primes:

Two numbers are said to be co-primes if their H.C.F. is 1.

H.C.F. AND L.C.M. OF FRACTIONS:

1. H.C.F. = $\frac{\text{H.C.F of numerators}}{\text{L.C.M of denominators}}$

2. L.C.M. = $\frac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}}$

H.C.F. AND L.C.M. OF DECIMAL FRACTIONS

In given numbers, make the same number of decimal places by annexing zeros in same numbers, if necessary. Considering these numbers without decimal point, find H.C.F or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

Illustrations –6:

Find out the H.C.F of 3^5, 3^9 and 3^{14}.

Solution:

Here the based of each number is same (=3) but indices are different.

So, the required H.C.F. = number with the minimum index, ie., 3^5.

Illustrations –7:

Find out the L.C.M. of 4^5, 4^8, 4^{12} and 4^7.

Solution:

Here the base of each number is the same (= 4) but indices (or powers) are different.

So, the required L.C.M. = number with the maximum index, i.e., 4^{12}.

Illustrations –8:

Find the least number exactly divisible by 12, 15, 20 and 27.

Solution:

Required number = L.C.M. of 12, 15, 20, 27.

3	12 – 15 – 20 - 27
4	4 – 5 – 20 - 9
5	1 – 5 – 5 - 9
	1 – 1 – 1 - 9

$\therefore$ L.C.M. $= 3 \times 4 \times 5 \times 9 = 540.$

Hence, required number = 540.

Illustrations –9:

H.C.F. and L.C.M. of two numbers are 16 and 240 respectively. If one of the numbers is 48, find the number.

Solution:

We know that, H.C.F × L.C.M. = Product of two numbers

$\therefore$ Second number = $\frac{16\times 240}{480}$ i.e., 80.

Illustrations –10:

Find the LCM and HCF of $\frac{25}{6}$ and $\frac{15}{4}$.

Solution:

LCM of $\frac{25}{6}$ and $\frac{15}{4}$

$= \frac{\text{LCM of } (25,15)}{\text{HCF of } (6,4)} = \frac{75}{2}$

HCF of $\frac{25}{6}$ and $\frac{15}{4}$ = $= \frac{\text{HCF of } (25,15)}{\text{LCM of } (6,4)} = \frac{5}{12}$

KEY POINTS

1. Suppose a number x divides a number y exactly. Then, we say that x is a factor of y. Also, in this case, we say that y is a multiple of x.
2. 1 is the only number having exactly one factor.
3. A number having exactly two factors is called a prime number.
4. The only even prime number is 12.
5. The HCF of two co-prime is 1.
6. If x is a factor of y then the HCF of x and y is x, and the LCM of x and y is y.
7. The HCF of two or more than two numbers is a factor of their LCM.
8. The product of the HCF and LCM of two number is equal to the product of the numbers.